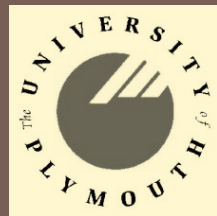


QUANTUM ELECTRODYNAMICS AT ULTRA-HIGH INTENSITIES

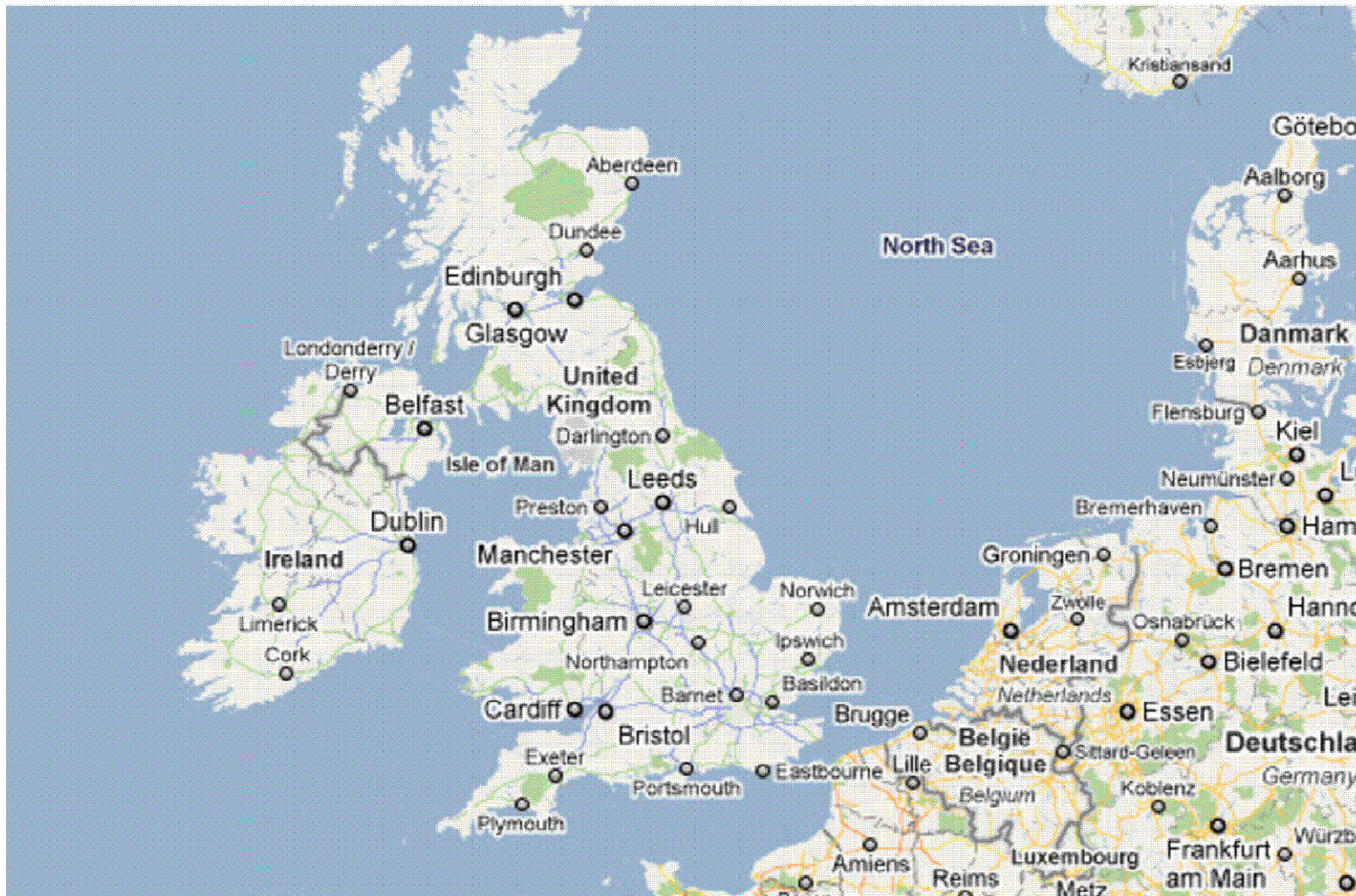
TOM HEINZL, UNIVERSITY OF PLYMOUTH, UK
RESEARCH PROGRESS MEETING, LBNL

30 JUNE 2011



With: N. Iji, K. Langfeld (UoP), C. Harvey, A. Ilderton, M. Marklund (Umeå), A. Wipf (Jena),
H. Schworer (Stellenbosch), B. Kämpfer, R. Sauerbrey, D. Seipt (FZD)

Plymouth? Portsmouth?



Outline

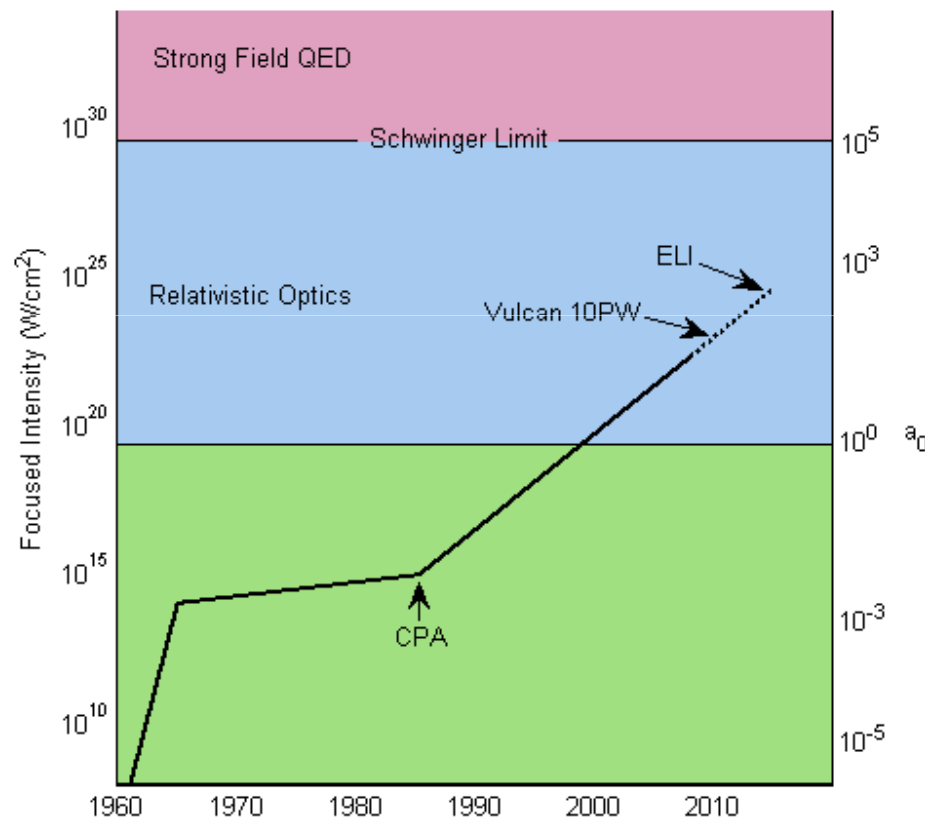


1. Introduction
2. Strong Laser Fields: Theory
3. Strong Laser Fields: Examples
 - A. Trees
 - B. Loops
4. Conclusion and Outlook



Introduction

50+1 Years of Laser Development



- Important parameter:
dim.less amplitude

$$a_0 \equiv \frac{eE\lambda}{mc^2} \sim I^{1/2}$$

- Energy gain of e^-
across laser wavelength
- $a_0 \gtrsim 1$: e^- relativistic

(adapted from Mourou, Tajima, Bulanov, RMP **78**, 2006)

Regime of Extremes

- Current magnitudes:

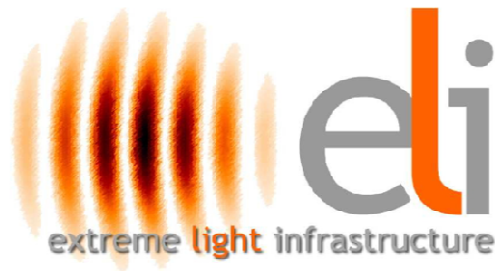
Power	$P \gtrsim 10^{15} \text{ W} \equiv 1 \text{ PW}$
Intensity	$I \gtrsim 10^{22} \text{ W/cm}^2$
Electric field	$E \gtrsim 10^{14} \text{ V/m}$
Magnetic field	$B \gtrsim 10^{10} \text{ G} \equiv 10^6 \text{ T}$

- Largest e.m. fields currently available in lab
- **But:** fields *pulsed* and *alternating*

2 Laser Projects (of many)



Building (projected)

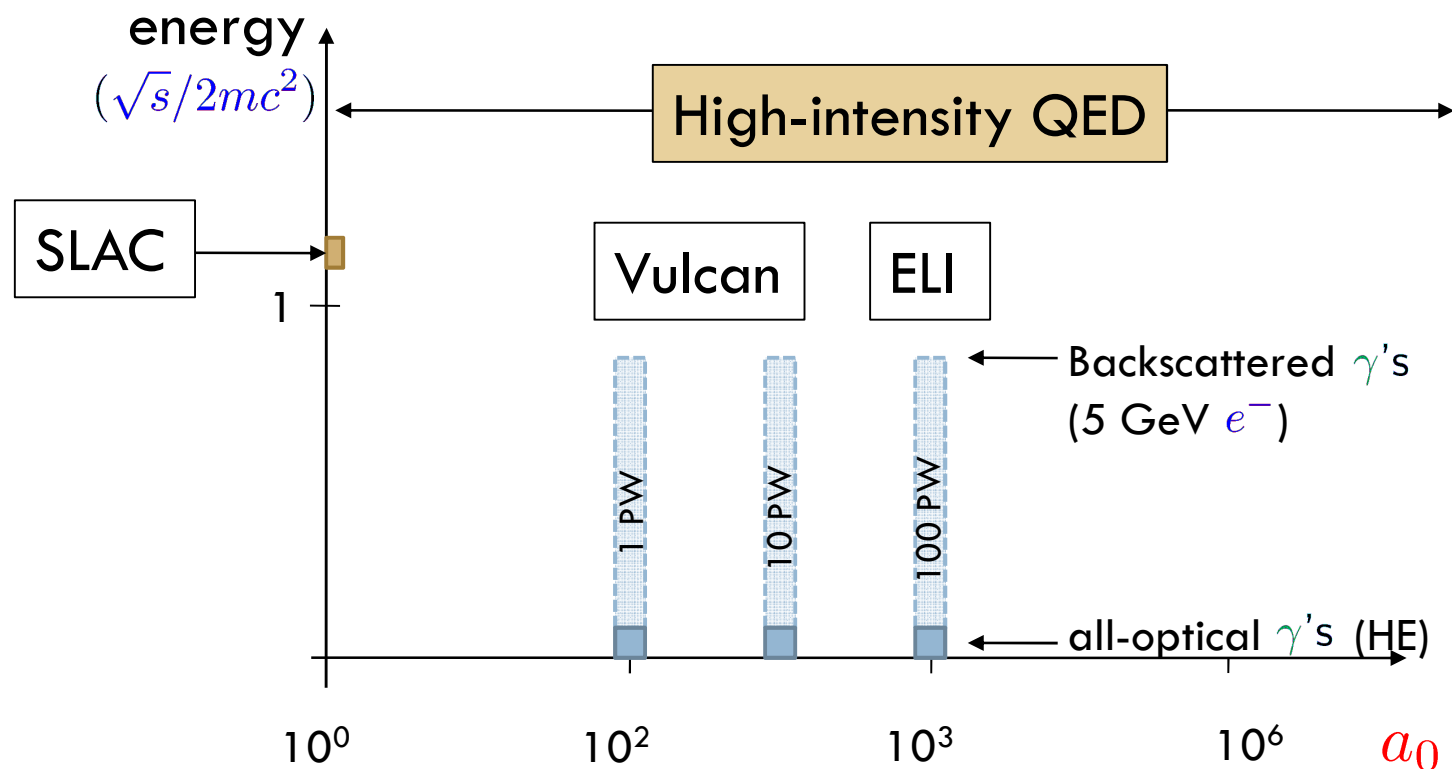


- CLF Vulcan 10 PW
 - 10^{23} Wcm^{-2}
 - Construction by 2014 (?)
 - Budget: 20 M£

- ELI ('4th pillar')
 - $>100 \text{ PW}$ (Exawatt ?)
 - $>10^{25} \text{ Wcm}^{-2}$
 - Budget: several 100 M€
 - Decision by 2012 (?)

Why bother?

- High intensity ($a_0 \gg 1$) = **uncharted** region of standard model (cf. phase diagrams)





2. Strong Laser Fields: Theory

Modelling a laser

□ In order of increasing complexity:

□ Plane wave



■ Infinite (IPW)

■ Pulsed (PPW)

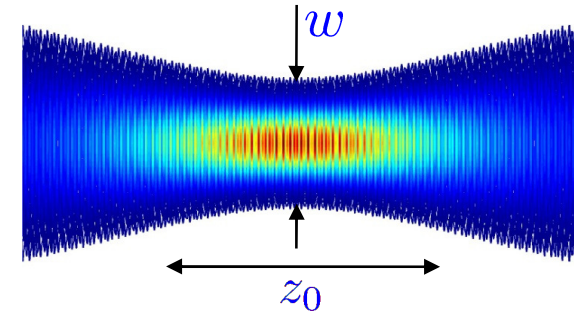
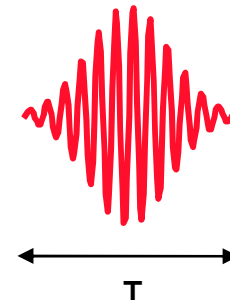
■ Finite T-duration

■ Infinite transverse extension


□ Gaussian beam:

■ Finite transverse waist w

■ Finite longitudinal extension z_0



Modelling a laser: Plane wave

- **Null** wave vector k , $k^2 = 0$ 
- Electromagnetic field $\mathbb{F} = (\mathbf{E}, \mathbf{B})$
 - only dependent on invariant phase $k \cdot x = \omega t/c - \mathbf{k} \cdot \mathbf{x}$
 - Transverse: $k\mathbb{F} = 0$
 - **Null:**
$$\mathcal{S} \equiv (E^2 - B^2)/2 = 0, \quad \mathcal{P} \equiv \mathbf{E} \cdot \mathbf{B} = 0, \quad \mathbb{F}^3 = 0$$
 - No intrinsic invariant scale!
 - Need (probe) momentum p to build invariants
 - E.g. $a_0 \sim \langle p, \mathbb{F}^2 p \rangle$ (TH, A. Ilderton, Opt. Comm. 2009)

Modelling a laser: Gaussian beam

- Finite geometry parameter:

$$\kappa \equiv w/z_0 \lesssim 1/2\pi$$

- PW limit: $\kappa \rightarrow 0$

- Transverse fields:

$$E_T = B_T \equiv E$$

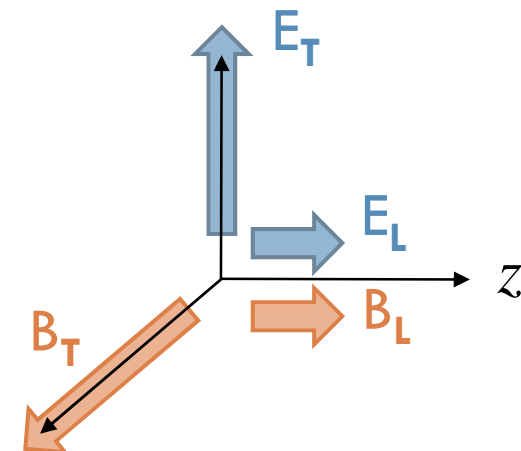
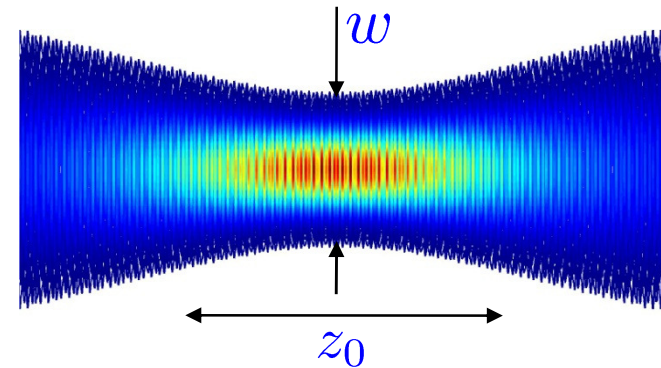
- Longitudinal fields:

$$E_L, B_L \sim \kappa E$$

- Invariants **not null** but $O(\kappa^2)$:

$$\mathcal{S} = (E_L^2 - B_L^2)/2, \quad \mathcal{P} = \mathbf{E}_L \cdot \mathbf{B}_L$$

(Davis 1978, Narozhny et al. 2004)



Charge in IPW

- Solution of Lorentz force eq.: rapid quiver motion (momentum $p(\tau)$)

- Charge acquires **quasi-momentum**

$$q \equiv \langle p \rangle = p_{\text{in}} + \kappa(a_0^2) k$$

- Longitudinal addition – consequence:

- **Effective mass squared**

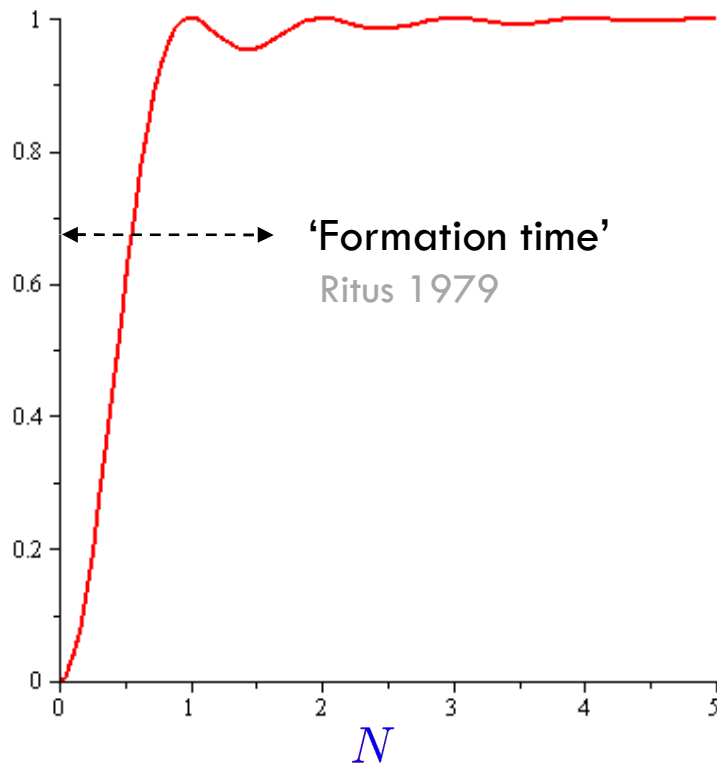
$$q^2 = m^2(1 + a_0^2) \equiv m_*^2$$

- *The basic intensity effect – yet unobserved so far!*

(Sengupta 1951, Kibble 1964)

Charge in PPW

$$a_0^2(N)/a_0^2(\infty)$$



Kibble, Salam, Strathdee 1975

□ a_0 = proper time average

- Mass shift Δm^2 depends on pulse duration, T
- gradually builds up with number N of cycles/pulse
- **Finite size effects**
(temporal & longitudinal)
- **NB:** ultra short pulses

$$T \sim O(1...10) \text{ fs}$$
$$N \sim O(1)$$

Charge in PW with RR

- Radiation Reaction: ever debated since Lorentz 1892

- ▣ Lorentz-Abraham-Dirac eq. → Landau-Lifshitz (LL) eq.

$$m\dot{u} = F + F_{\text{RR}} = F + \tau_0 \mathbb{P}\dot{F}$$

Lorentz force

$$g^{\mu\nu} - u^\mu u^\nu / c^2$$

- ▣ time parameter $\tau_0 \equiv 2e^2/3mc^3 \equiv 2r_e/3c \sim 10^{-23} \text{ s}$

- **Q:** Can one see RR?

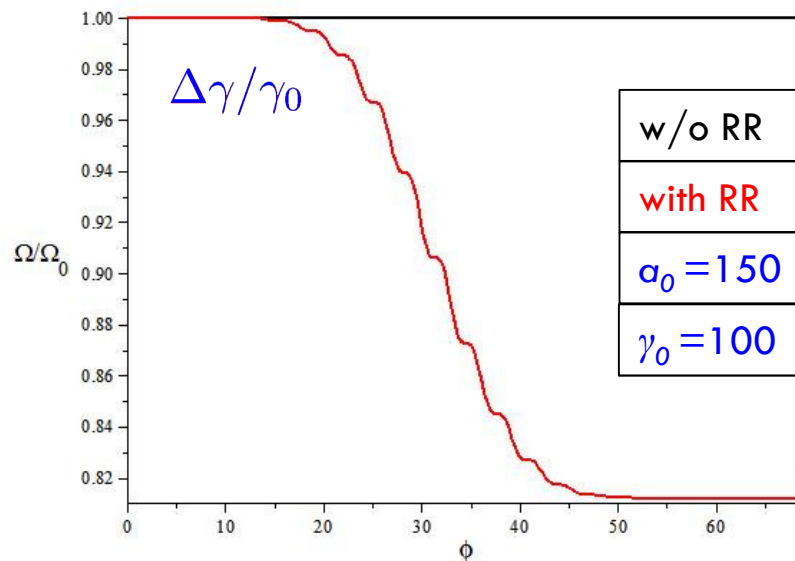
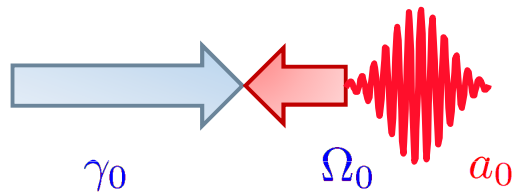
- Analytic solution for PW (Di Piazza, 2008, Harvey; TH, Iji, Langfeld 2011)

- ▣ RR important when $\Omega_0 \tau_0 \simeq 1$ (Ω_0 = frequency ‘seen’ by probe e^-)

- ▣ ‘fixed target mode’: $\Omega_0 \tau_0 \simeq 10^{-8}$; ‘colliding mode’: $\Omega_0 \simeq 2\gamma_0 \Omega_{\text{lab}}$

RR signature: energy loss in pulse

- Relative energy loss for head-on collision



- Solution of LL eq.

$$\Delta\gamma/\gamma_0 \simeq 1 - 2\pi N \Omega_0 \tau_0 a_0^2$$

- RR signal enhanced by

- pulse duration

$$2\pi N \gg 1$$

- intensity

$$a_0^2 \gg 1$$

- and Doppler upshift

$$\Omega_0 \simeq 2\gamma_0 \Omega_{\text{lab}}$$

Quantum parameters

□ Vacuum sector:

- 'critical' electric field (Sauter 1931, Schwinger 1951)

$$E_S \equiv \frac{m^2 c^3}{e \hbar} = 1.3 \times 10^{18} \text{ V/m}$$

- c and \hbar : relativity \cup quantum mechanics: **QED**

□ Charge sector:

- Laser energy as seen by probe electron

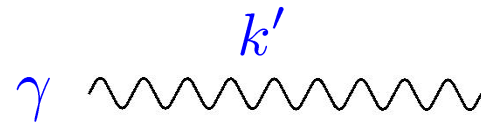
$$\nu_0 \equiv \frac{\hbar k \cdot u}{m c^2} = \frac{\hbar \Omega_0}{m c^2}$$

- **NB:** not necessarily small (Doppler shift !)

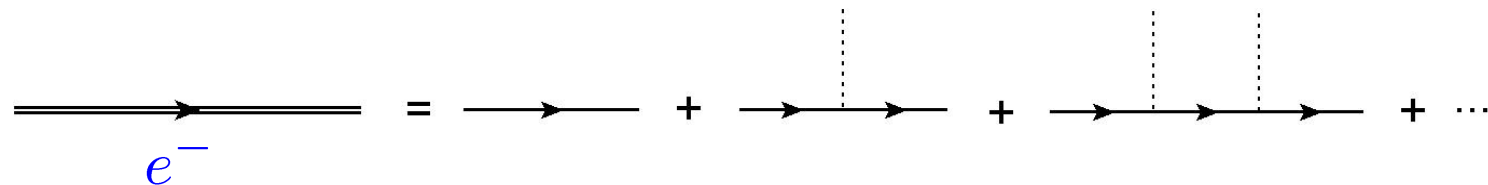
Strong-field QED

Ingredients:

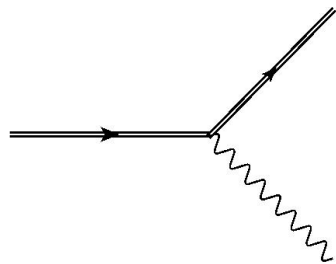
Probe photons



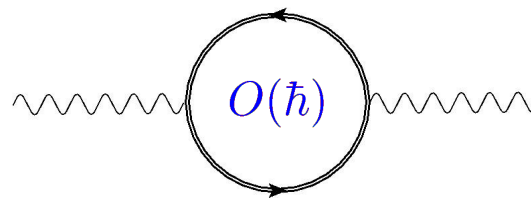
Electrons 'dressed' by laser photons (-----)



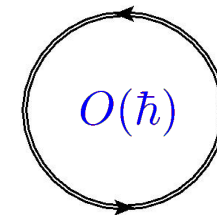
“Furry Picture” Diagrams:



Scattering



Vacuum polarisation



Vac \rightarrow Vac

Main issues

- Intensity dependence of elementary processes (see below ☒)
- Finite (beam) size effects (see below ☒)
- Beyond plane waves (? ☐)
- Classical vs. quantum (including RR) (? ☐)



3. Strong Laser Fields: Examples

A. Trees

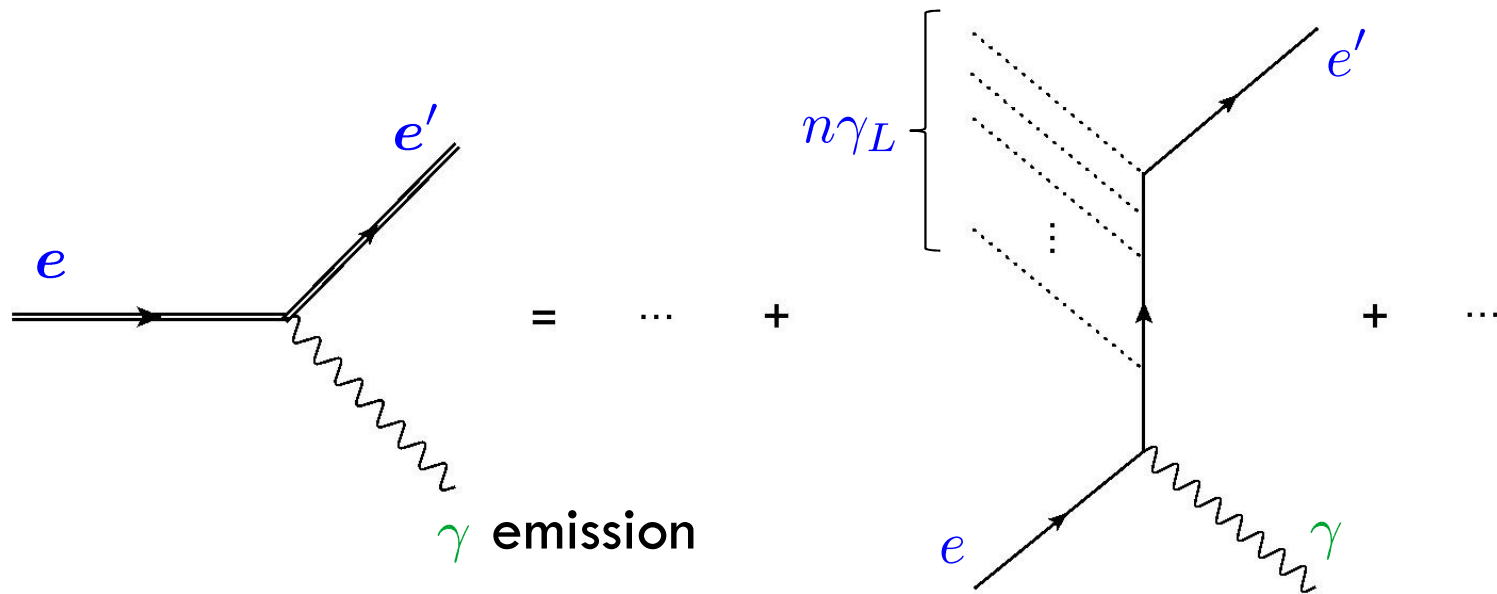


3. Strong Laser Fields: Examples

3.1 Nonlinear Compton Scattering (NLC)

NLC scattering

- Expand Furry picture Feynman diagram \rightarrow
- Sum over all processes of the type $e + n\gamma_L \rightarrow e' + \gamma$

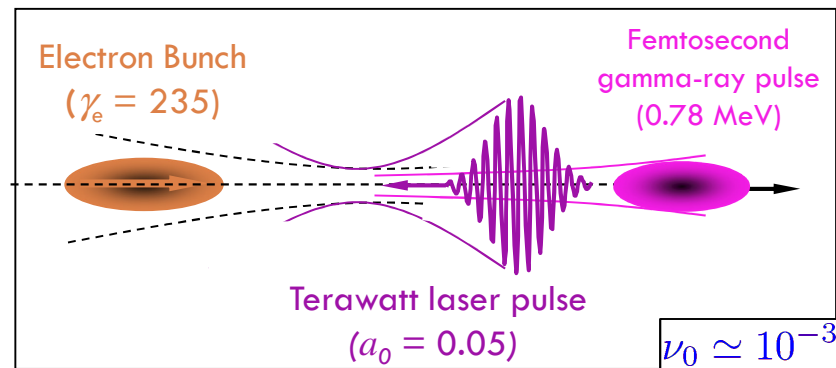


Schott 1912; Nikishov/Ritus 1964,
Brown/Kibble 1964, Goldman 1964

NLC: main features

- No energy threshold – can be done **now!**
- Classical limit: NL Thomson ($\nu_0 \ll 1$ or $2\gamma_e \hbar \omega \ll mc^2$)
- For $a_0 < O(1)$: frequency upshift $\omega'_{\max} \simeq 4\gamma_e^2 \omega$

- Used for
X-ray generation



T-REX, LLNL (2008)

- Nonlinearity:

$$N_\gamma \sim \sigma(a_0) N_e N_{\gamma_L}$$

NLC cont^d

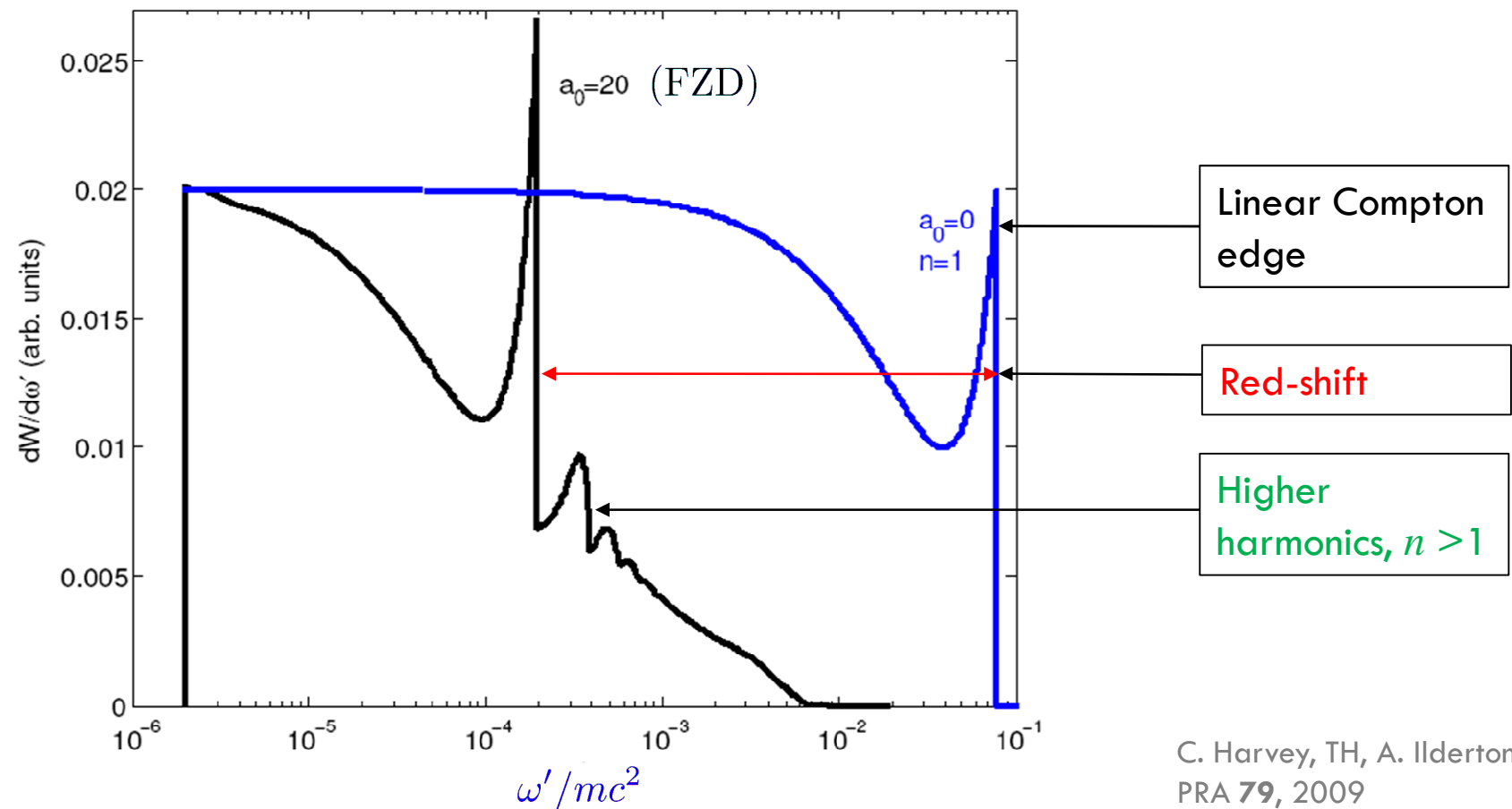
- For high intensity, $a_0 \gg 1$
- **modified** Compton edge due to mass shift

$$\omega'_{n,\max} \simeq 4\gamma_e^2 n\omega / a_0^2, \quad n = 1, 2, \dots$$

- In particular:
 - ▣ Higher harmonics: $n > 1$ (Chen, Maksimchuk, Umstadter, Nature 1998)
 - ▣ Overall blueshift maintained as long as $a_0 \lesssim 2\gamma_e$
 - ▣ **Redshift** of $n=1$ edge

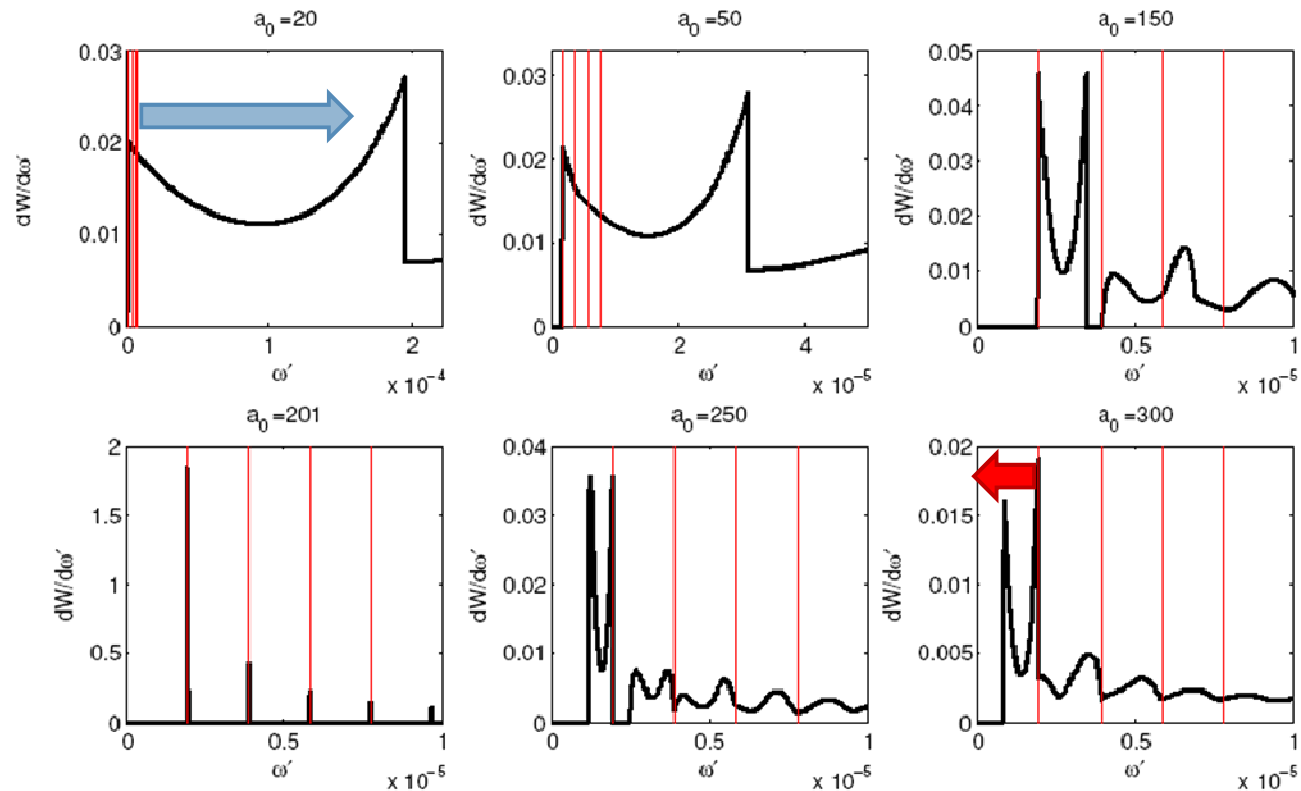
$$\omega'_{\max} \simeq 4\gamma_e^2 \omega \longrightarrow 4\gamma_e^2 \omega / a_0^2$$

NLC spectrum: main a_0 effects



C. Harvey, TH, A. Ilderton,
PRA **79**, 2009

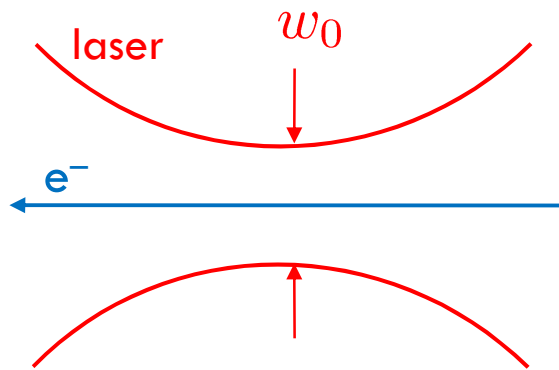
a_0 dependence (lab)



Tuning a_0 similar to changing frame: when $a_0 = a_{0c} \simeq 2\gamma$
 ‘inverse’ Compton \rightarrow Compton

Finite Size Effects

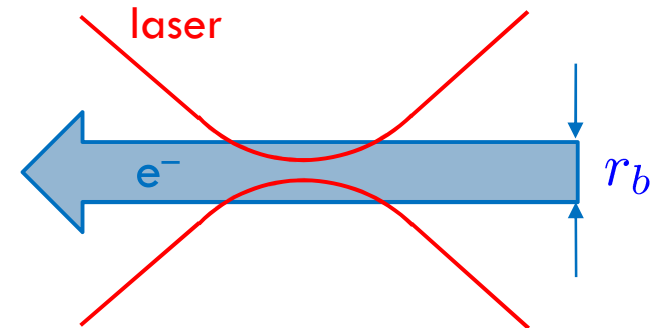
Weakly focussed: $w_0 \gg r_b$



$$a_0 = O(1)$$

PW results 'realistic'

Strongly focussed: $w_0 < r_b$



$$a_0 \gg 1$$

PW results get modified

NLC vs. RR (in progress)

- **Linear Thomson:** modified by RR (Dirac 1938, Heitler 1941, Gora 1943)

$$\sigma_{RR} \simeq \sigma_{Th} \left(1 - \frac{4}{9} \alpha^2 \nu_0^2\right)$$

- Compare with **NLC**

$$\sigma_{NLC} \simeq \sigma_{Th} \left(1 - \underset{\substack{\uparrow \\ \text{QM}}}{2\nu_0}\right) \left(1 - \frac{2}{5} \underset{\substack{\uparrow \\ \text{NL}}}{a_0^2}\right)$$

- Hence:

- RR must be classical limit of **higher order radiative correction** (IR photons?)
- Is there classical regime where RR gets boosted by a_0^2 (cf. LL eq.) ?

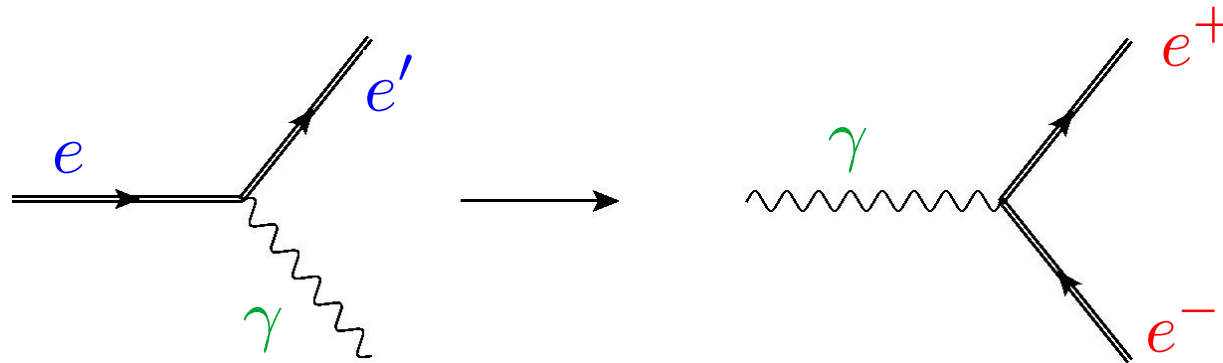


3. Strong Laser Fields: Examples

3.2 Laser Induced Pair Production (PP)

Stimulated PP

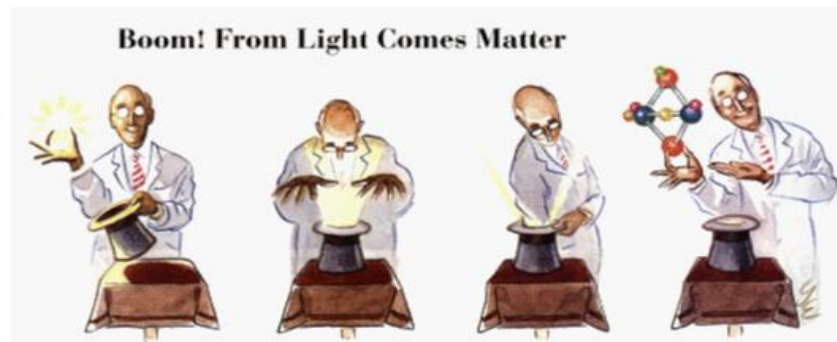
- Obtained from NLC via crossing



- Main new feature: threshold $2m_*^2$
- Experiment SLAC E-144 (1995): combine both processes @ high energies ($2\gamma_e \hbar \omega \simeq mc^2$)
- \rightarrow Quantum regime...

SLAC E-144 (Bula et al. '96, Burke et al. '97)

- Two stages: $e + n\gamma_L \rightarrow e' + \gamma$ NLC
 $\gamma + n\gamma_L \rightarrow e^+ + e^-$ stimulated PP



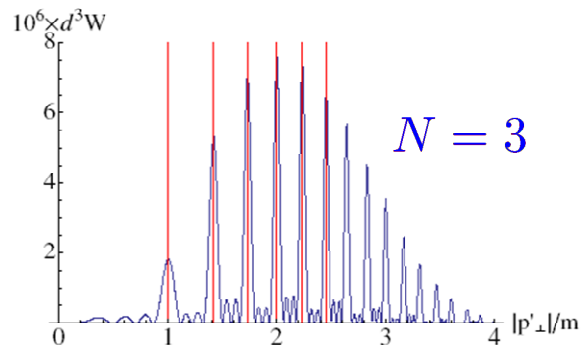
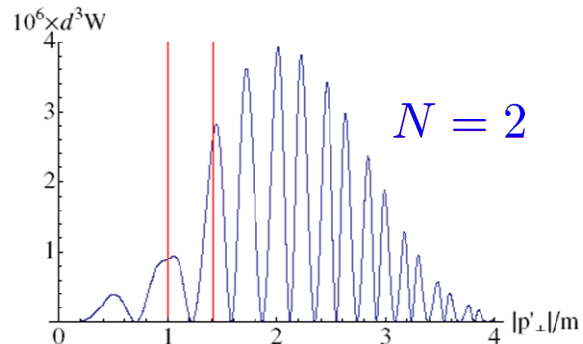
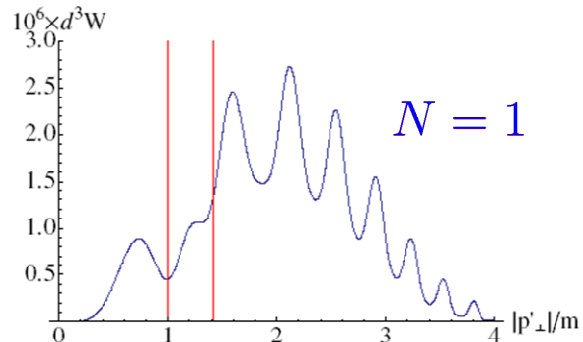
Gil Eisner, Photonics Spectra 1997

$$50 \text{ GeV } e^- \rightarrow 30 \text{ GeV } \gamma \rightarrow O(10^2) \text{ pairs} \quad \left[\begin{array}{l} a_0 \simeq 0.5 \\ n = 5 \end{array} \right]$$

- New development: prediction of pair cascades

(Bell, Kirk et al.; Narozhny, Fedotov, Ruhl et al.)

Stimulated PP: finite-size effects



□ IPW:

- triple-diff rate = ‘delta comb’
- above threshold (m_*)

□ PPW:

- dependence on cycles per pulse, N
- **Sub-threshold** signals
- IPW approached for $N \gg 1$



3. Strong Laser Fields: Examples

B. Loops



3. Strong Laser Fields: Examples

3.3 Vacuum Birefringence (VB)

Heisenberg, Euler 1936

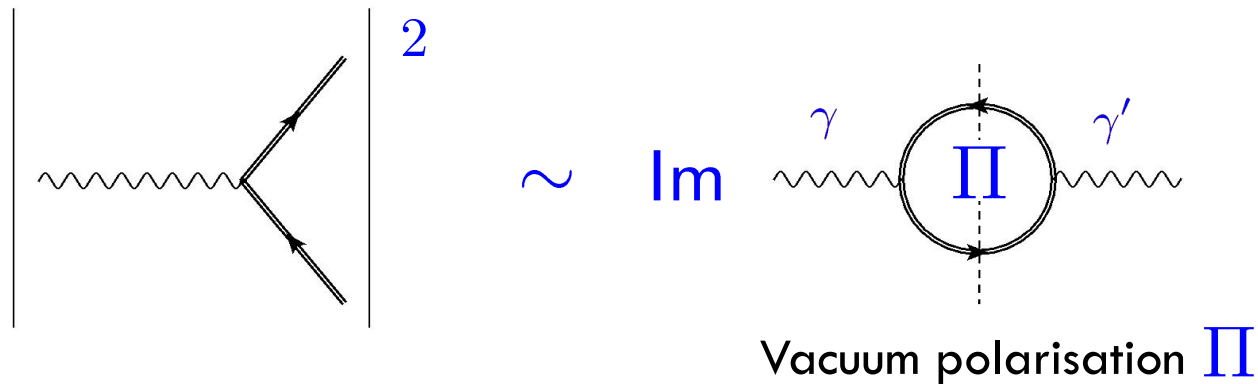


andererseits wird selbst dort, wo die Energie zur Materieerzeugung nicht ausreicht, aus ihrer virtuellen Möglichkeit eine Art „Polarisation des Vakuums“ und damit eine Änderung der Maxwell'schen Gleichungen resultieren.

“...even in situations where the [photon] energy is not sufficient for matter production, its virtual possibility will result in a ‘polarization of the vacuum’ and hence in an alteration of Maxwell’s equations.”

Optical Theorem (Trees \rightarrow Loops)

- Total PP rate can be obtained via



The diagram illustrates the optical theorem. On the left, a wavy line (photon) enters from the left and splits into two fermion lines (solid lines with arrows) that exit to the right. This is enclosed between two vertical lines. To the right of this diagram is a blue superscript '2'. This is followed by a tilde '~' and the text 'Im'. To the right of 'Im' is a loop diagram labeled with a blue Π. The loop is a circle with two arrows indicating a clockwise flow. A wavy line labeled γ enters from the left, and a wavy line labeled γ' exits to the right. Below the loop diagram is the text 'Vacuum polarisation Π'.

- Virtual $e^+ e^-$ 'dipoles' feel presence of **E**
- Re Π : change of polarisation state $\gamma \rightarrow \gamma'$
- diagonalisation of Π (for X-fields = $\text{PW}_\omega \rightarrow 0$)
- **two** nontrivial eigenvalues \rightarrow

Vacuum birefringence (Brezin, Itzykson 1970)



Calcite crystal

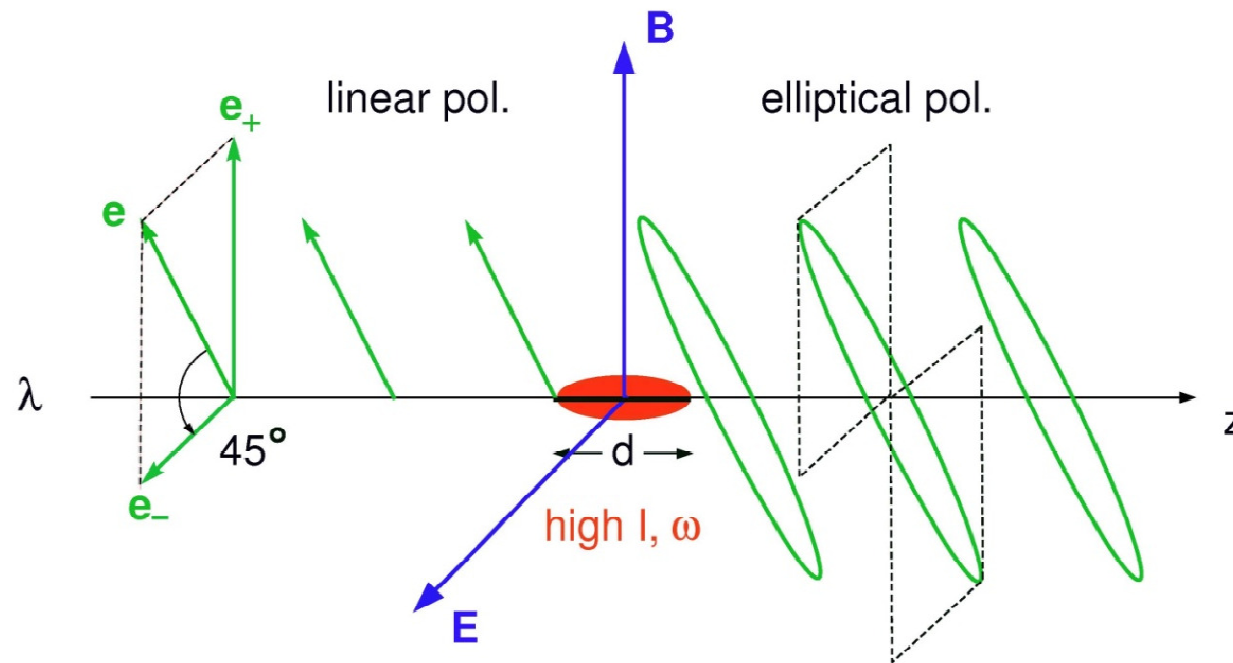
- Two indices of refraction (Toll 1952)

$$n_{\pm} = 1 + \frac{\alpha \epsilon^2}{45\pi} \{11 \pm 3 + O(\epsilon^2 \nu^2)\} \{1 + O(\alpha \epsilon^2)\}$$

- Dim.less (small) parameters:

- ▣ Field strength: $\epsilon \equiv E/E_S$
- ▣ Probe frequency: $\nu \equiv \hbar\omega/mc^2$
- ▣ fine structure const: $\alpha = 1/137$

Experiment: measure ellipticity



Phase retardation of e_+

Analysis (TH et al., Opt. Comm., 2006)

- ellipticity (squared)

$$\delta^2 = 3.2 \times 10^5 \left(\frac{d}{\mu\text{m}} \epsilon^2 \nu \right)^2, \quad \epsilon \nu \ll 1$$

- Power law suppressed...

- Optimal scenario @ ELI

- ▣ large intensity: $\epsilon \simeq 10^{-2}$

- ▣ large probe frequency (X-ray, $\nu \simeq 10^{-2}$):

$$\delta^2 \simeq 10^{-7} \dots 10^{-4}$$

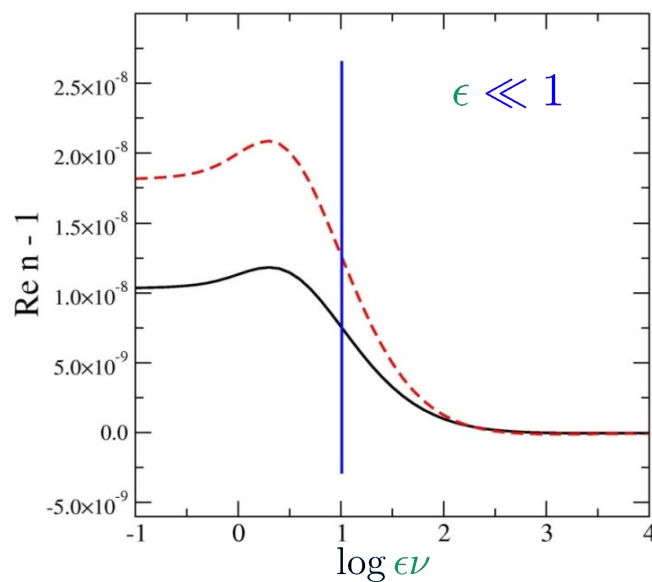
- ▣ New record in polarisation purity: 1.5×10^{-9} @ 6 keV

(Marx et al., Opt. Comm., 2010)

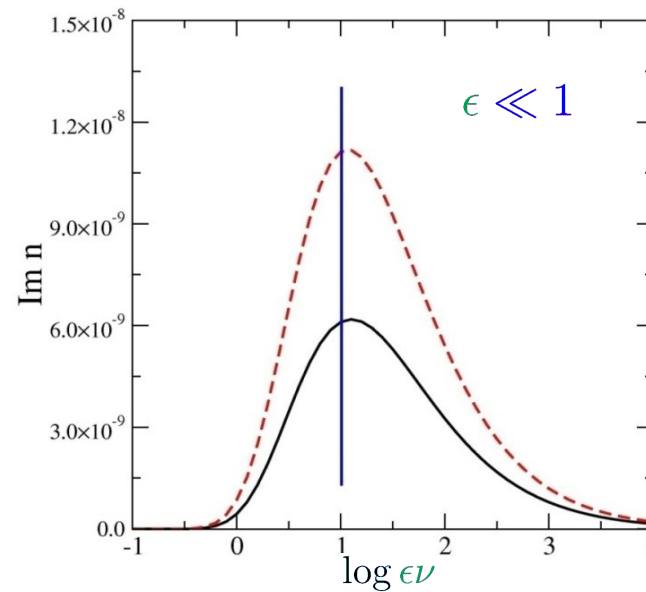


Large- ν birefringence via NLC

- $\epsilon\nu \simeq 3$ for e^- : 3 GeV @ ELI, 10 GeV @ Vulcan10PW



Anomalous dispersion



Absorption \rightarrow PP

(Toll 1952
TH, O. Schröder
2006
Shore 2007)

(K. Langfeld)

- NB: SLAC E-144 had $\epsilon\nu \simeq 0.1$



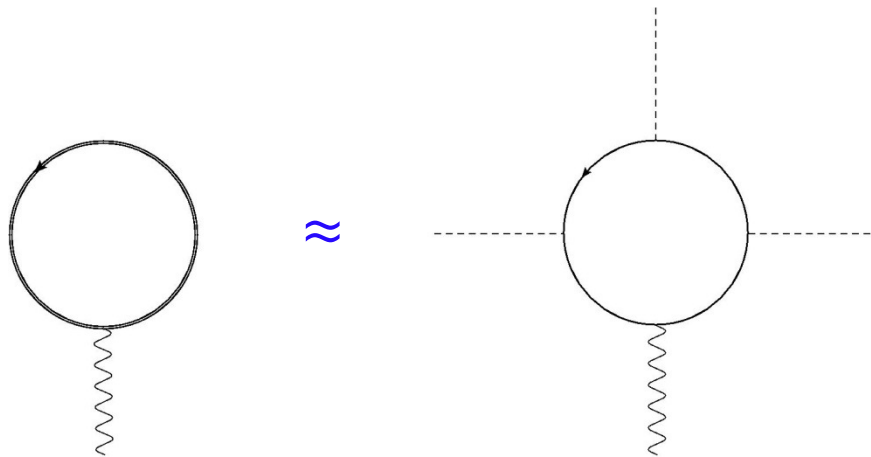
3. Strong Laser Fields: Examples

3.3 Light-by-Light Scattering

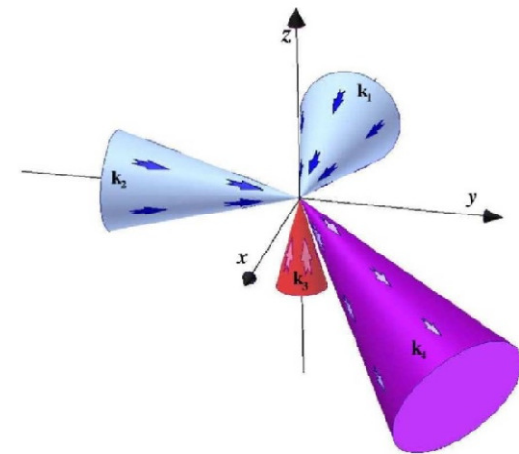
γ - γ scattering

- Predicted: 1930's (Halpern 1934, Euler/Kockel 1935, Euler/Heisenberg 1936)
- But never observed in lab!
- Idea: $3\gamma_L \rightarrow \gamma$ (Lundström et al. 2005)

Feynman diagrams:



Artistic view:



γ - γ scattering cont^d

- Low-energy X-section (Euler-Heisenberg approxⁿ):

$$\sigma_{\gamma\gamma} = \frac{973}{10125\pi^2} \alpha^2 r_e^2 \nu_L^6 \simeq 10^{-67} \text{ cm}^2$$

- Laser photon density:

$$n_L \simeq 10^{14} a_0^2 / \mu\text{m}^3$$

- Photon number in focus volume $(10 \mu\text{m})^3$

$$N_\gamma \simeq 10^{17} a_0^2$$

- Number of emitted γ 's @ $a_0 \simeq 10^2$

$$N_{\gamma'} \simeq \frac{\sigma_{\gamma\gamma}}{(10 \mu\text{m})^2} N_\gamma^3 \simeq 10^2$$



3. Strong Laser Fields: Examples

3.4 Vacuum Pair Production

Spontaneous (vacuum) PP

- Feynman diagram

$\text{Im} \quad \text{[Feynman diagram: a circle with a vertical dashed line through its center]} \sim \left| \text{[Feynman diagram: a semi-circle with an arrow pointing left, labeled } e^+ \text{ at the top and } e^- \text{ at the bottom]} \right|^2$ ‘vacuum breakdown’

- Identically **zero** for PWs as $\mathcal{S} = \mathcal{P} = 0$
- Substantial when

$$E_0 \equiv \left(\sqrt{\mathcal{S}^2 + \mathcal{P}^2} + \mathcal{S} \right)^{1/2} \gtrsim E_S$$

- Rate exponentially suppressed (Schwinger 1951)

$$\mathfrak{R} \sim \exp(-\pi E_S / E_0)$$

Vacuum PP cont^d

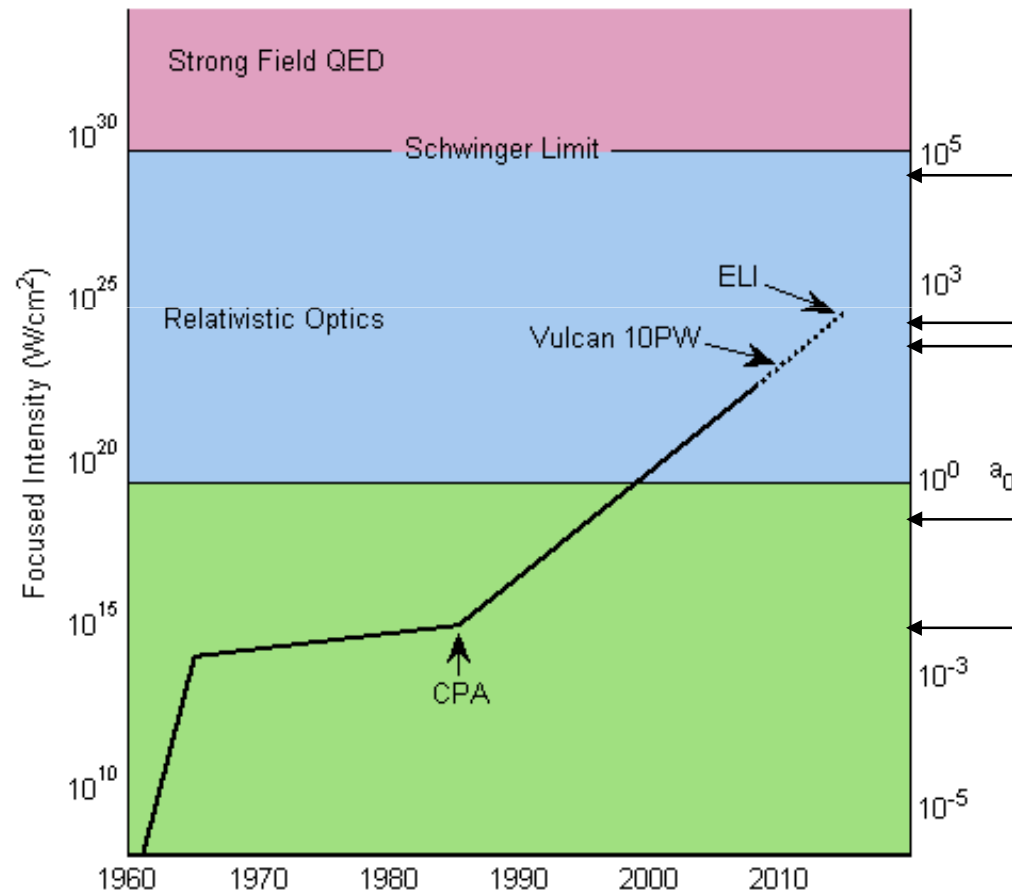
- With lasers: **very difficult!**
- Need to fight both
 - ▣ Exponential suppression
 - ▣ Null field (plane wave) character
- Expect rate for e.g. Gaussian beams

$$\mathfrak{R} \sim \kappa^2 \coth(\pi B_L / E_L) \exp(-\pi E_S / \kappa E_T)$$

- Alternative: counter-propagating lasers (standing wave)?

$$\mathcal{S} \neq 0 \quad \text{and/or} \quad \mathcal{P} \neq 0$$

Summary



vacuum PP

$$\text{vac} \rightarrow e^+ + e^-$$

$\gamma\gamma$ scattering

$$3\gamma_L \rightarrow \gamma'$$

Vacuum birefringence

$$\gamma \rightarrow \gamma'$$

Stimulated PP

$$\gamma + n\gamma_L \rightarrow e^+ + e^-$$

NL Compton/Thomson

$$e + n\gamma_L \rightarrow e' + \gamma$$

Conclusion

- Laser power approaching exawatt regime
- Extreme field physics @ low energy
 - ▣ Lab astrophysics
 - ▣ New physics (axions, hidden photons, ... ?)
 - ▣ **Laser QED** → Sauter-Schwinger limit
- Theory (→ a_0 dependence + signatures)
 - ▣ Challenges:
 - Finite size effects
 - Beyond plane waves
 - Numerical approaches
 - Radiation reaction: Classical vs. quantum



Thank you very much...

...for your attention

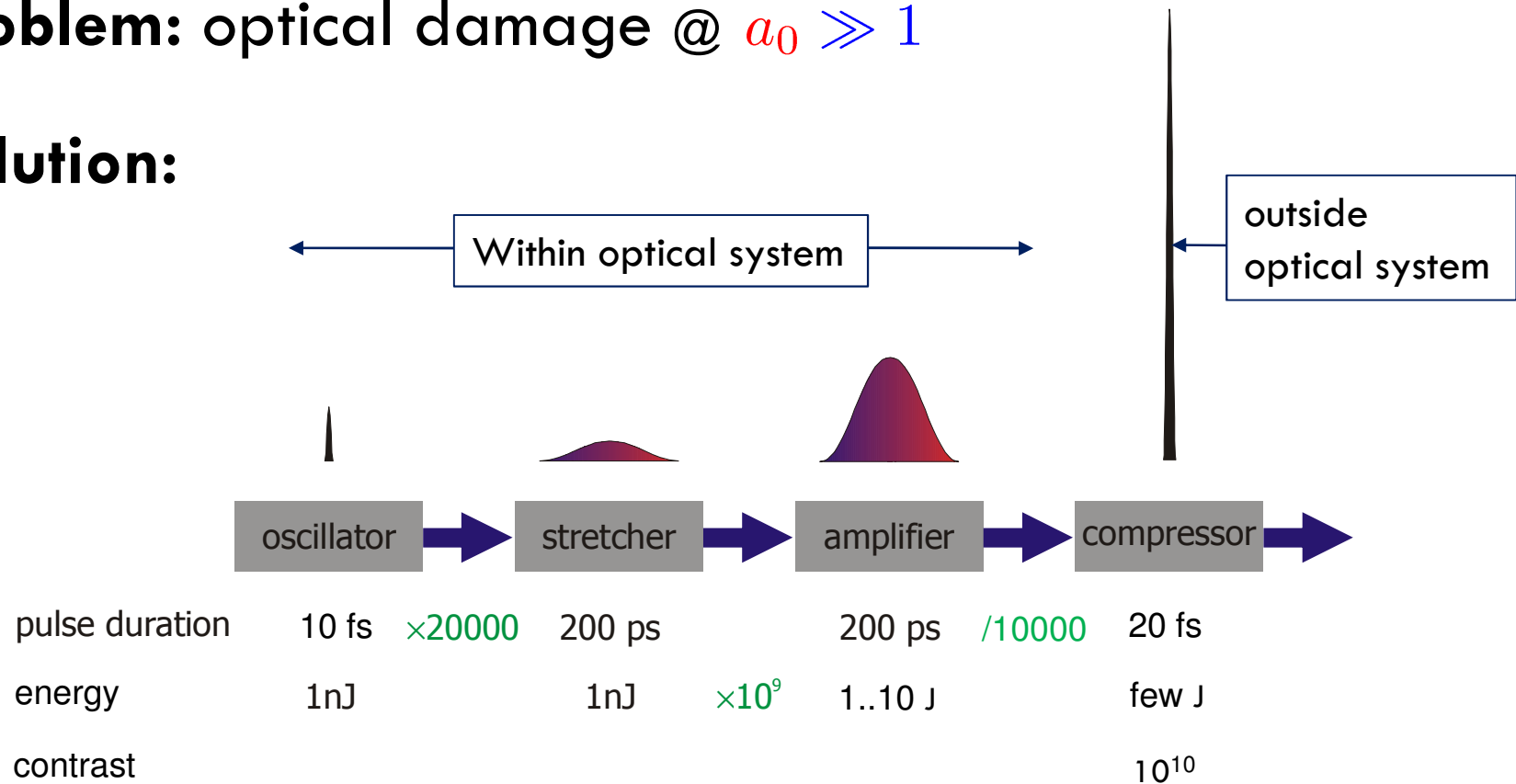


Appendix

Chirped Pulse Amplification (CPA)

Problem: optical damage @ $a_0 \gg 1$

Solution:



Courtesy R. Sauerbrey

4 cases of fields (Taub 1948)

□ Table:

Name	Special frame (SF)	Invariants	
Electromagnetic	$\mathbf{E} \parallel \mathbf{B}$	$\mathcal{P} \neq 0$	$\mathcal{S} \neq 0$
Magnetic	\mathbf{B}	$\mathcal{P} = 0$	$\mathcal{S} < 0$
Electric	\mathbf{E}	$\mathcal{P} = 0$	$\mathcal{S} > 0$
Null	$\mathbf{E} \perp \mathbf{B}, \mathbf{E} = \mathbf{B}$	$\mathcal{P} = 0$	$\mathcal{S} = 0$

Remarks on a_0

$$a_0 \equiv \frac{e \sqrt{-\langle \mathbf{E} \cdot \mathbf{E} \rangle} \lambda_0}{m c^2}$$

- $-\mathbf{E} \cdot \mathbf{E} \equiv (u, \mathbb{F}^2 u)/c^2$: energy density ‘seen’ by e^-
- $\langle \dots \rangle$: proper time average (see below)
- For non-periodic fields (pulses):

$$\langle f(\tau) \rangle = \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} d\tau f(\tau) , \quad (\text{Kibble et al. 1975})$$

$$\langle \mathbf{E} \cdot \mathbf{E} \rangle \rightarrow \text{var}(\mathbf{E})$$

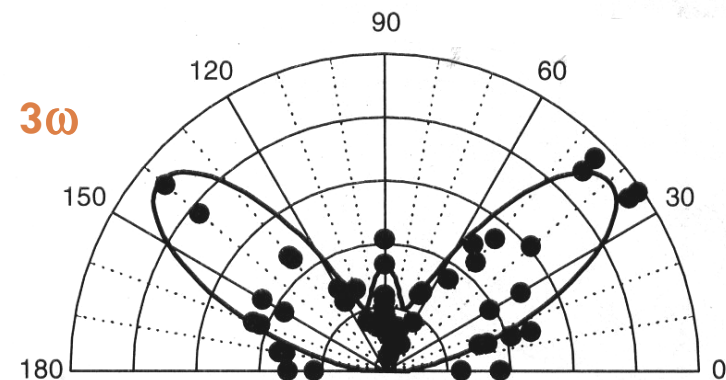
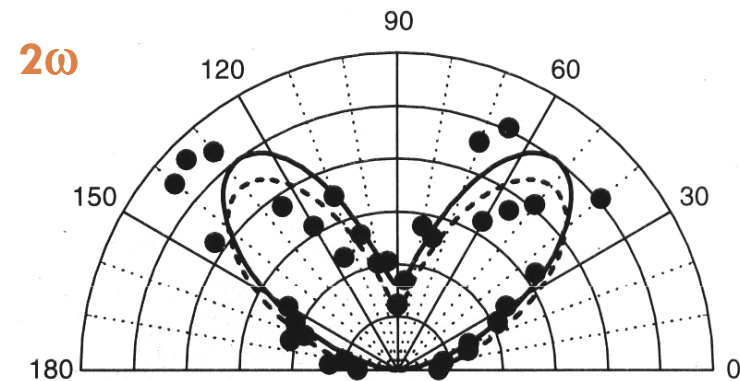
- **Note:** a_0 is **not** a vacuum field characteristic

Aside: Higher harmonics

- Harmonics $n=2$ and $n=3$ observed in ‘relativistic Thomson scattering’ using *linearly* polarised laser ($a_0=1.88$)

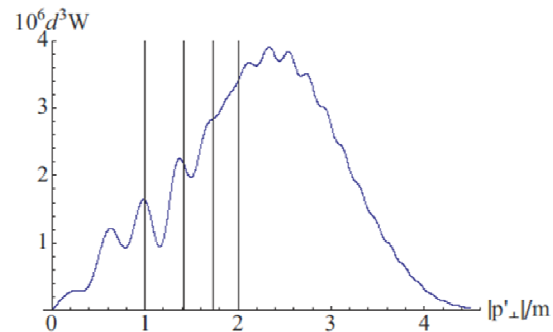
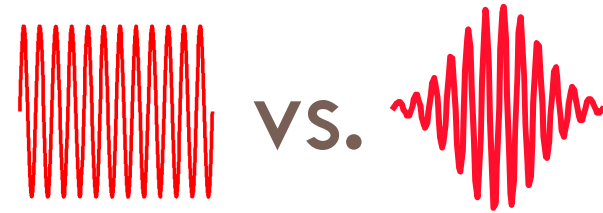
- Signal: quadrupole and sextupole pattern in angular distribution

(Chen, Maksimchuk, Umstadter, Nature 1998)

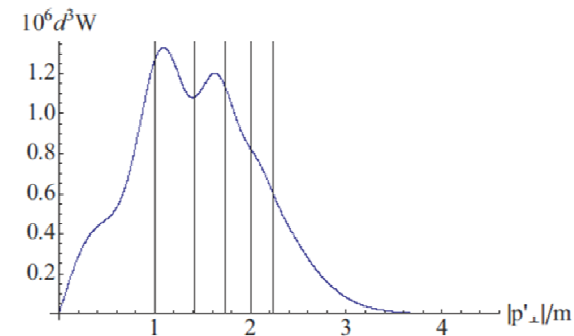


$$\theta = 90^\circ$$

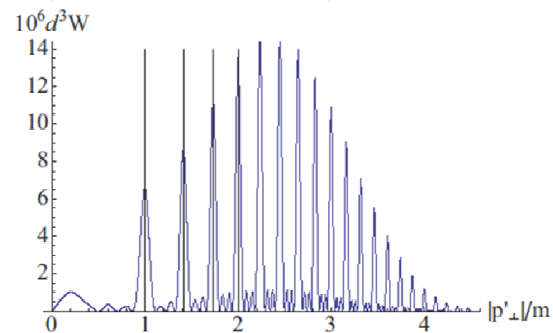
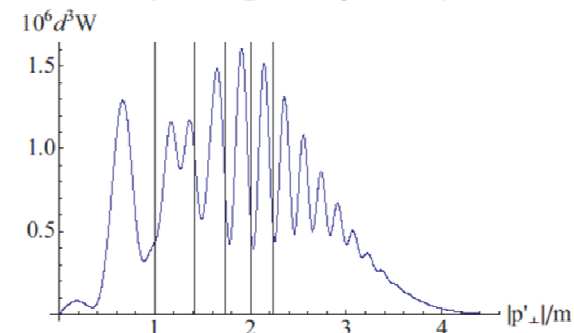
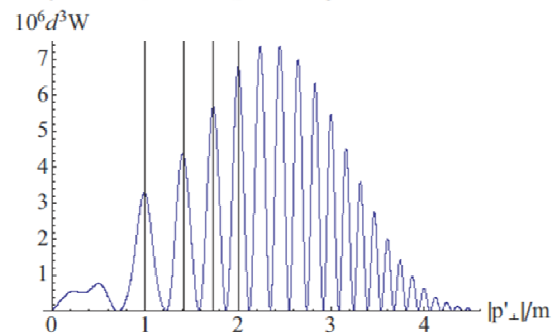
Wave train vs. pulse:



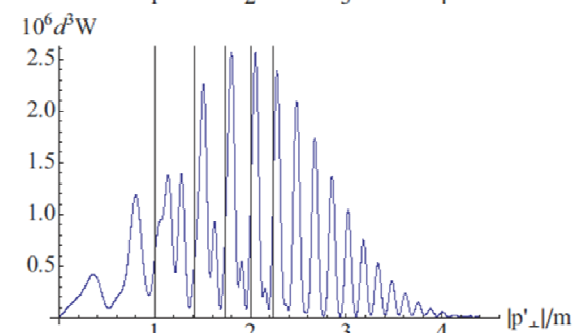
$N = 1$



$N = 4$



$N = 8$



→ Spectrum = fingerprint of pulse!